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SURFACE INTERMEDIATE ZONE OF SUBMERGED TURBULENT BUOYANT JET IN CURRENT

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ABSTRACT

This paper deals with the intermediate zone between the jet and plume stages of a submerged buoyant discharge from sea outfall in current. The stability criteria, plume width and height after the intermediate zone and the dilution within the intermediate region have been studied theoretically and experimentally. The study indicates that a stable upstream wedge may be existed when the densimetrical Froude number (F_{Δ}) is less than 0.45, an unstable wedge is likely to appear when F_{Δ} is between 0.45 ~ 1.0, and no upstream wedge exists once F_{Δ} is greater than 1.0; the plume height and width could be readily estimated and finally the dilution in the intermediate zone is found to be insignificant.

1. INTRODUCTION

A turbulent buoyant jet discharged into a homogeneous flowing ambient is illustrated in Fig. 1. Basically, this process can be classified into three stages, namely jet, intermediate and plume stages.

In the jet stage, the discharged flow is mainly governed by the jet momentum and buoyancy, and the self-generated turbulence plays a dominating role in the path and dilution of the jet; In the plume stage, the plume is chiefly controlled by the buoyancy and the ambient turbulence.

Between the jet and plume stages, there exists a transition zone called intermediate stage/zone. In the intermediate stage, the effluent flow is governed by both momentum and buoyancy. It appears to be very difficult to describe the distribution of velocity and concentration in details in the intermediate stage because of the fact that both buoyancy and remained momentum will cause the jet to move horizontally with radial velocities as soon as the jet reaches the water free surface. Generally speaking, the buoyancy is intending to stabilize the upstream wedge and the cross flow momentum is to destructure the upstream intrusion.

In the past, the studies on turbulent buoyant jets and plumes in current have been mainly focused on the jet and plume zones, Larsen(1994) and Larsen,Petersen & Chen(1990), a little attention has been payed to the intermediate zone which links the jet and plume stages as a bridge.

Cederwall (1971) studied the stability of the upstream wedge using a two layer flow system which is essentially similar to saline intrusion at estuary. The flow was classified into three types, namely supercritical, subcritical and jet/plume like flows using the momentum flux ration and the source Froude number. A plant submerged buoyant jet with arbitrary angle into a stagnant, shallow fluid was studied theoretically and experimentally by Jirka & Harlemam (1973). The flow regions were divided into submerged, surface impingement, hydraulic jump and stratified counterflow regions. The stability and mixing characteristics were investigated. A round vertical buoyant jet in still water was studied by Lee & Jirka (1981). The same principles were used for the 2-D case with two differences, (a) a detailed treatment of the zone of flow establishment and (b) the transition from the surface impingement region to the outer flow is taken as a combination of a radial surface jet and a hydraulic jump.

In summary of the previous studies, it seems that the studies carried out were mainly concentrating on stagnant receiving waters except Cederwall's study, but Cederwall's study included only the stability analysis.

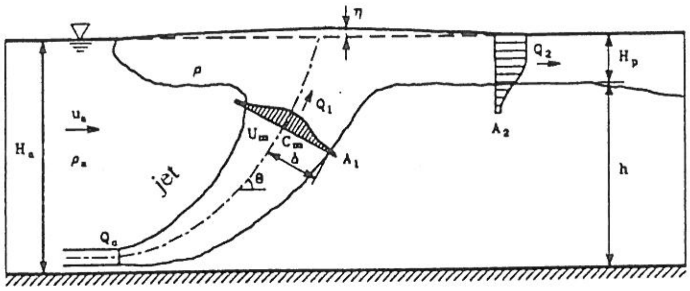
For a buoyant jet with arbitrary angle into a flowing ambient, in the author's view, in the stage of intermediate, three main aspects are of interests to be discussed. Firstly, the stability criterion for the upstream wedge induced by a turbulent submerged buoyant jet; secondly the initial plume height and width for the plume stage, and thirdly, the dilution in the intermediate zone.

The objective of this study is to investigate the intermediate stage covering the three aforementioned aspects, in order to shed some lights on this largely neglected area and later to develop a model which is able to describe the whole process in one code, including the jet, intermediate and plume stages.

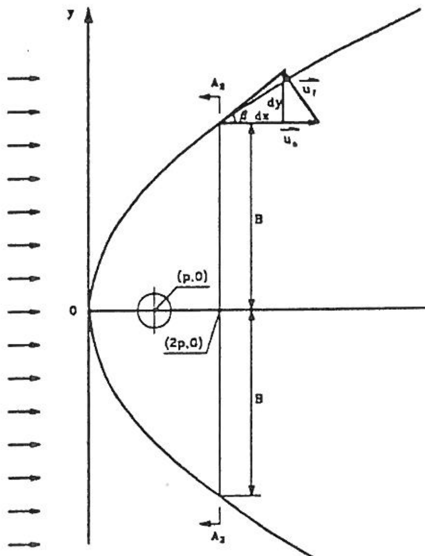
2. THEORY

2.1. Stability Criteria for Upstream Wedge

An upstream wedge created by a submerged turbulent buoyant jet in a homogenous flowing ambient may be formed on the water surface and sketched in Fig. 1.



(a) Side View



(b) Plan View

Fig. 1. Schematic Diagram of upstream wedge

It is assumed that the Bernoulli equation applies to a surface and a bottom streamline, Fig.1(a), the following equations are obtained immediately, Hansen & Jensen (1981).

$$\eta = \frac{u_a^2}{2g}$$

$$H_a + \frac{u_a^2}{2g} = h + \rho\frac{H_p}{\rho_a} + \frac{u_a^2}{2g}\left(\frac{H_a}{h}\right)^2 \tag{1}$$

in which, η : surface excess water lever, u_a : ambient current velocity, H_a : ambient water depth; h : water depth under the plume; ρ_a : ambient density; ρ : plume density; g : gravitational acceleration;

since $H_a+\eta=h+H_p$ and $\eta / H_a << 1$, then $h/H_p \approx 1-h_p/H_a$, thus, the Bernoulli equation finally yields:

$$2\left(1-\frac{H_p}{H_a}\right)^2\frac{H_p}{H_a} = \frac{u_a^2}{\frac{\Delta\rho}{\rho_a}gH_a} \tag{2}$$

In fact the right hand side of Eq.(2) is a densimetrical Froude number,the maximum value of which is 0.296 for H_p/H_a in the range of 0.0 ~ 1.0,therefore, the criterion to form a stable upstream wedge depends on the value of the densimetrical Froude number, in other words, a stable upstream wedge may be established if

$$F_{\Delta s} = \frac{u_a^2}{\frac{\Delta\rho}{\rho_a}gH_a} \le 0.296 \approx 0.30 \tag{3}$$

The above analysis of stability condition for the upstream wedge is based on the two-layer flow system without taking the shape or the width of the surface plume into account.

Another approach to establish the stability criterion for the upstream wedge and to find the height of the plume is to consider the front velocity driven by buoyancy on the surface, Schrøder(1990), see Fig. 1.(b).

$$tg\beta = \frac{u_f}{\sqrt{u_a^2-u_f^2}} \tag{4}$$

in which, the front velocity u_f can be expressed as

$$u_f = \left(1-\frac{H_p}{H_a}\right)\sqrt{\frac{\Delta\rho}{\rho_a}gH_p} \tag{5}$$

substituting Eq.(5) into Eq.(4), one obtains:

$$tg^2\beta\left[u_a^2-\left(1-\frac{H_p}{H_a}\right)^2\frac{\Delta\rho}{\rho_a}gH_p\right] = \left(1-\frac{H_p}{H_a}\right)^2\frac{\Delta\rho}{\rho_a}gH_p \tag{6}$$

where $tg\ \beta = \sqrt{2}/2$ by assuming the shape of the surface plume as a parabolic and written as $y^2 = 4px$, defining the location of the width of the plume at $x = 2p$, where p is the focal distance of the parabolic. Rearranging Eq.(6), it yields

$$3\left(1-\frac{H_p}{H_a}\right)^2\frac{H_p}{H_a} = \frac{u_a^2}{\frac{\Delta\rho}{\rho_a}gH_a} \tag{7}$$

This leads to a similar equation to Eq.(3) with a different factor of 3 instead of 2. Hence, the criterion for a stable upstream intrusion created by a submerged buoyant jet in current states that the densimetrical Froude number has to be smaller than 0.45 instead of 0.30.

2.2. Plume Height and Width

In a stable condition, the plume height can be calculated from either Eq.(2) or Eq.(7).

The relation between the plume height and width can be derived by applying the continuity equation of mass in the intermediate zone. By substituting the Gaussian profile of velocity and integrating across the sections A_1 and A_2, the continuity equation of mass reads:

$$\iint_{A_1} [\ u_m \exp(-\frac{r^2}{b^2}) + u_a \cos\theta\ ]\ dA = \iint_{A_2} u_a\ dA \tag{8}$$

After the integration, it becomes,

$$\pi b^2(\ u_m + 2u_a \cos\theta\ ) = 2u_a B H_p \tag{9}$$

Finally, the half width of the plume is found to be

$$B = \frac{\pi b^2(u_m + 2u_a \cos\theta)}{2u_a H_p} = \frac{S_o Q_o}{2u_a H_p} \tag{10}$$

in which, S_o is the initial dilution defined at the end of the jet stage and Q_o is the initial jet discharge rate at the nozzle.

2.3. Dilution in Intermediate Zone

The conservation equation of density deficiency in the intermediate stage can be written as:

$$\frac{d}{ds}[\iint_A u\ \Delta\rho\ dA] = 0 \tag{11}$$

Substituting the Gaussian profiles of velocity and density, then integrating across the sections A_1 and A_2, Eq.(11) becomes:

$$\pi\lambda^2 b^2[\frac{u_m}{1+\lambda^2} + u_a \cos\theta]\Delta\rho_{m1} = \frac{\pi}{2} u_a \Delta\rho_{m2} B H_p \tag{12}$$

Finally, the dilution S_2 in the intermediate stage can be estimated as:

$$S_2 = \frac{\Delta\rho_{m1}}{\Delta\rho_{m2}} = \frac{u_a H_p B}{2\lambda^2 b^2(\frac{u_m}{1+\lambda^2} + u_a \cos\theta)} \tag{13}$$

It is believed that the excess jet velocity (u_m) is negligible at this stage in comparison to the ambient velocity and the angle θ is less than 45 degrees in the presence of the ambient current in most cases. Therefore it is perhaps reasonable to assume that $u_m \approx 0$ and $\cos\theta \approx 1.0$, then Eq.(13) can be approximated as

$$S_2 = \frac{\Delta\rho_{m1}}{\Delta\rho_{m2}} \approx \frac{u_a H_p B}{2b^2\lambda^2 u_a} \approx \frac{2}{5}\frac{H_p B}{b^2} \tag{14}$$

by taking the spreading coefficient $\lambda = 1.16$. The dilution could be estimated in the order of 2 ~ 4 times based on the assumption that the plume height H_p has the same order of magnitude as the jet radius b and the plume width B is about 5 ~ 10 times of b.

3. EXPERIMENT

Laboratory experiments were carried out in a flume with a dimension of 20 meter long, 1.5 meter wide and 0.8 m deep. Heated water discharged from a nozzle located at 10 cm above the flume bed and both flesh and salt receiving waters were used, The detailed experiment set-up and results are provided in Chen (1991) and Chen,Larsen & Petersen(1991).

3.1. Stability of Upstream Wedge

A series of experiments were performed in order to verify the theoretical stability criteria derived for the upstream wedge using both dye observation and photographic technique. It has been found in the laboratory studies that a stable upstream wedge is formed under the condition that the surface plume densimetrical Froude number ($F_{\Delta s}$) is less than 0.45 which coincides with the theory. The upstream wedge created by the submerged turbulent buoyant jet will be completely expired if $F_{\Delta r} > 1.0$ and an unstable upstream wedge is likely to appear when $F_{\Delta r}$ is in the range of $0.45 \sim 1.0$. It seems that the flow regimes found in the experiment could be categorized into three types with stable, unstable and no upstream wedges.

3.2. Plume Height and Width

The experimental data on plume heights and widths are analyzed by calculating the variances of the measured cross-sections:

$$\sigma_y^2 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}(y-\bar{y})^2 \Delta\rho_{ij}\Delta y_i \Delta z_j}{M_o} \qquad \sigma_z^2 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} z^2 \Delta\rho_{ij}\Delta y_i \Delta z_j}{M_o} \tag{15}$$

in which,

$$M_o = \sum_{i=1}^{m}\sum_{j=1}^{n}\Delta\rho_{ij}\Delta y_i \Delta z_j \qquad \bar{y} = \sum_{i=1}^{m}\sum_{j=1}^{n} y_i \Delta\rho_{ij}\Delta y_i \Delta z_j \tag{16}$$

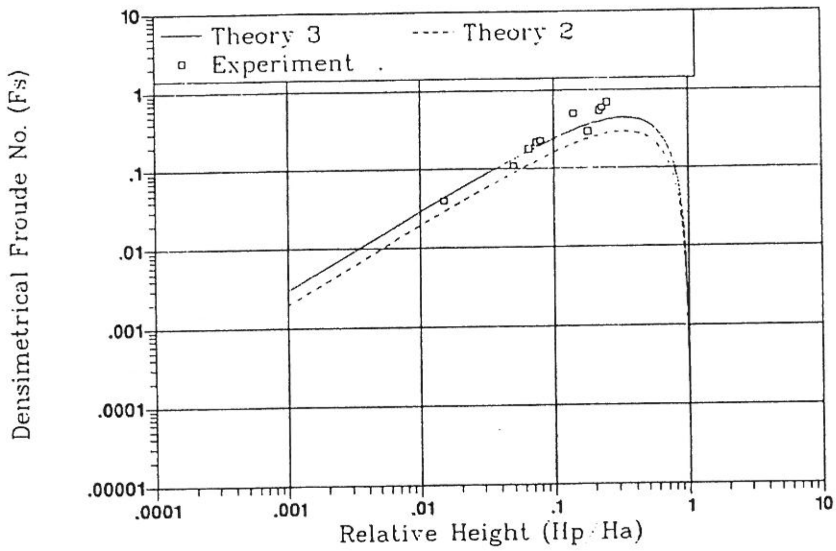


Fig. 3. Plume Height after Intermediate Zone

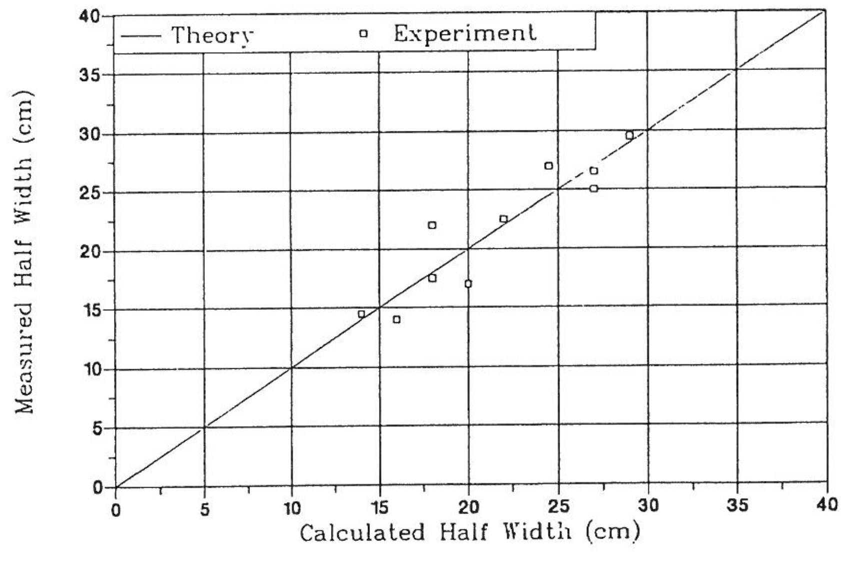


Fig. 4. Plume Width after Intermediate Zone

In order to ensure the measurements correct the total mass budget is checked against the initial discharged mass by comparing the terms of $\iint_{A_0}\Delta\rho_0 u_0 dA$ and the zeroth moment M_o. Theoretically, the following equation should be valid:

$$\iint_{A_0} \Delta\rho_o u_o \, dA = \iint_{A} \Delta\rho u_a \, dy \, dz \qquad (17)$$

based on the assumption that the temperature T, and density difference obey a linear relation, i.e. $\Delta\rho_o = \beta_a \Delta T$, where, β_a is a constant.

The experimental data on the plume heights are plotted in Fig. 3 and compared with the theory. It shows that the experimental data agree better with the Theory 3 (factor of 3) derived by applying the front velocity driven by buoyancy force than the Theory 2 (factor of 2) based on the Bernoulli equations. It also indicates that the data in the stable region ($F_{\Delta r} < 0.45$) fit better with the theories than that in the unstable region ($0.45 < F_{\Delta r} < 1.0$).

The experimental data on plume width are demonstrated in Fig. 4 and compared with the theory. It seems that the measured widths are reasonably well correlated with the calculated widths. It should be pointed out that the dilution in the theory is the average dilution defined as $S_o = Q/Q_o$ but in the analysis of the experimental data on the plume width the minimum centerline dilution $S_{min} = \Delta\rho_o/\Delta\rho_m$ was used instead.

4. CONCLUSION

The intermediate zone of a turbulent buoyant jet in flowing ambient has been investigated theoretically and experimentally, the following conclusions may be draw from this study:

A stable upstream wedge could be formed if the densimetrical Froude number ($F_{\Delta r}$) defined as Eq. (3) is less than 0.45; an unstable upstream wedge may be appeared when the densimetrical Froude number is in the range of $0.45 \sim 1.0$ and no upstream wedge exists when $F_{\Delta r}$ is greater than 1.0.

With a stable upstream wedge, the plume height and width at the end of the intermediate zone could be readily estimated using Eq.(7) and Eq.(10), respectively.

The dilution in the intermediate zone is found to be insignificant and estimated to be in an order of magnitude of $2 \sim 4$ times, provided that the upstream wedge is relatively stable.

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